# Crystal Structure of $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ 

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The crystal structure of $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ has been determined from single-crystal three-dimensional x-ray diffraction data. The monoclinic crystal has cell dimensions $a=12.23 \pm 0.02, b=3.04 \pm$ $0.01, c=5.80 \pm 0.01 \mathrm{~A}$ and $\beta=103.7 \pm 0.3^{\circ}$ as originally reported by Kohn, Katz, and Broder [Am. Mineral. 42, 398 (1957)]. There are $4 \mathrm{Ga}_{2} \mathrm{O}_{3}$ in the unit cell. The most probable space group to which the crystal belongs is $C_{2 h^{3}}{ }^{3}-C 2 / m$; the atoms are in five sets of special positions $4 i:\left(000, \frac{1}{2} \frac{1}{2} 0\right) \pm(x 0 z)$. There are two kinds of coordination for $\mathrm{Ga}^{3+}$ ions in this structure, namely tetrahedral and octahedral. Average interionic distances are: tetrahedral $\mathrm{Ga}-\mathrm{O}, 1.83 \mathrm{~A}$; octahedral $\mathrm{Ga}-\mathrm{O}, 2.00 \mathrm{~A}$; tetrahedron edge $\mathrm{O}-\mathrm{O}, 3.02 \mathrm{~A}$; and octahedron edge $0-\mathrm{O}, 2.84 \mathrm{~A}$. Because of the reduced coordination of half of the metal ions, the density of $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ is lower than that of $\alpha-\mathrm{Ga}_{2} \mathrm{O}_{3}$ which has the $\alpha$-corundum structure. Also the closest approach of two $\mathrm{Ga}^{3+}$ ions in $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ is 3.04 A which is considerably larger than the closest approach of metal ions in the sesquioxides with the $\alpha$-corundum-type structure and, in agreement with the results of thermodynamic meas-
urements, the $\beta$ phase appears to be the structurally more stal one.

The average $\mathrm{Ga}-\mathrm{O}$ distances in the structure seem to accou for the fact that although the $\mathrm{Ga}^{3+}$ ion is substantially larg than the $\mathrm{Al}^{3+}$ ion its quantitative preference for tetrahedrally c ordinated sites when substituted for $\mathrm{Fe}^{3+}$ ion in the iron garne is very nearly the same as that of the $\mathrm{Al}^{3+}$ ion.

The structure accounts for a recent result obtained by Pet and Schawlow from paramagnetic-resonance measurements $\mathrm{Cr}^{3+}$-ion-doped $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$, namely that the $\mathrm{Cr}^{3+}$ ion substitutes the $\mathrm{Ga}^{3+}$ ions in a single set of equivalent octahedral sites.

The magnetic aspects of the $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ structure are discussed ar it is shown that a possible $\mathrm{Fe}_{2} \mathrm{O}_{3}$ isomorph could be expected to at least antiferromagnetic with a Néel temperature of about $700^{\circ}$ Furthermore, a knowledge of the $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ structure and of t nature of site preferences of the $\mathrm{Ga}^{3+}$ and $\mathrm{Fe}^{3+}$ ions in the garne lead to a prediction regarding the structure of the ferrimagnet crystals of formula $\mathrm{Ga}_{2-x} \mathrm{Fe}_{x} \mathrm{O}_{3}$ recently discovered by Remeika

## INTRODUCTION

MUCH attention has recently been given the sesquioxides of the $3 d$ elements particularly in regard to their magnetic nature. Several of these have the $\alpha-\mathrm{Al}_{2} \mathrm{O}_{3}$ ( $\alpha$-corundum) structure. ${ }^{1}$ Crystals of $\alpha-\mathrm{Al}_{2} \mathrm{O}_{3}$ doped with small amounts of magnetic ions are not only of scientific interest but have important application as MASER materials. Trivalent gallium with $3 d^{10}$ configuration is nonmagnetic and therefore a $\mathrm{Ga}_{2} \mathrm{O}_{3}$ crystal doped with a magnetic ion is also of scientific and possibly technological interest.

In these Laboratories paramagnetic-resonance studies by Peter and Schawlow ${ }^{2}$ on $\mathrm{Cr}^{3+}$-ion-doped $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ crystals prepared by Remeika ${ }^{3}$ indicated that the $\mathrm{Cr}^{3+}$ ions had replaced $\mathrm{Ga}^{3+}$ ions in crystallographically equivalent octahedral sites. If the $\mathrm{Cr}^{3+}$ ions had entered crystallographically nonequivalent octahedral sites, this certainly would have been detected. On the other hand, it would not have been simple to detect tetrahedrally coordinated $\mathrm{Cr}^{3+}$ ions.

Since Kohn, Katz, and Broder had shown ${ }^{4}$ that the unit cell of $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ contained $8 \mathrm{Ga}^{3+}$ ions, then, as will be shown in detail later, these ions must be in at least two sets of crystallographically nonequivalent positions. Because there seemed little reason to expect the $\mathrm{Cr}^{3+}$

[^0]ions to prefer one set of octahedral sites to another an because the $\mathrm{Cr}^{3+}$ ion shows an exclusive preference ff octahedral coordination, (see, for example, footnot references 5-7) it appeared reasonable to speculate th: the $\mathrm{Ga}^{3+}$ ions had two types of coordination in $\mathrm{Ga}_{2} \mathrm{O}_{3}$. If this speculation proved sound, we would hav the first clear-cut case in which a pure sesquioxic contained octahedrally and tetrahedrally coordinate metal ions in the same structure.

With the present interest in the $3 d$ transition-met. oxides, it would seem worthwhile to determine t structures of any of those yet unknown. (Indeed the now appears to be good reason to refine known structur of these oxides to provide needed accuracy for theoret cal considerations.) Such knowledge could be importar to our understanding of the crystal chemistry of th ions involved. For example, the $\mathrm{Fe}^{3+}$ and $\mathrm{Ga}^{3+}$ ions wit spherical electronic configurations $3 d^{5}$ and $3 d^{10}$, r spectively, have both octahedral and tetrahedr: coordination in the garnets $\left\{\mathrm{X}_{3}\right\}\left[\mathrm{Fe}_{2}\right]\left(\mathrm{Fe}_{3}\right) \mathrm{O}_{12}$ an $\left\{\mathrm{Y}_{3}\right\}\left[\mathrm{Ga}_{2}\right]\left(\mathrm{Ga}_{3}\right) \mathrm{O}_{12}$ and in solid solutions of one these in the other. ${ }^{5}$ Both the $\mathrm{Ga}^{3+}$ and $\mathrm{Al}^{3+}$ ions a: smaller than the $\mathrm{Fe}^{3+}$ ion, but the $\mathrm{Ga}^{3+}$ is substantial! larger than the $\mathrm{Al}^{3+}$ ion. ${ }^{8}$ Yet when these ions are sul stituted for the $\mathrm{Fe}^{3+}$ ion in the garnets they show vet nearly the same quantitative preference for tetrahedr sites. ${ }^{5,9}$ The present investigation appears to clarit this observation.

The structure of $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ also accounts for its beir

[^1]the thermodynamically stable room-temperature phase, ${ }^{10}$ for its having a lower density than that of the $\alpha$ phase, and allows us to make a speculation regarding the structure of the recently discovered ferrimagnetic (and piezoelectric) $\mathrm{Ga}_{2-x} \mathrm{Fe}_{x} \mathrm{O}_{3}{ }^{3}{ }^{3}$

## DETERMINATION OF THE STRUCTURE

The crystals used in this study were prepared by J. P. Remeika. ${ }^{3}$ Powdered specimens gave photographs in agreement with the patterns reported by Kohn ct al. ${ }^{4}$ and by Foster and Stumpf ${ }^{10}$ for the $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$. Most of the crystals photographed were twinned, some multiply, but after some searching one was found from which a small single-crystal segment with long direction along the monoclinic $b$ axis was cleaved. The cell con$\operatorname{stants}{ }^{11} a=12.23 \pm 0.02, b=3.04 \pm 0.01, c=5.80 \pm 0.01 \mathrm{~A}$, $\beta=103.7 \pm 0.3^{\circ}$ given by Kohn et al. ${ }^{4}$ are quite accurate for our purposes as established by the complete indexing of the powder pattern taken with $\mathrm{CuF} x$ radiation (see Appendix). The space group to which 1 e crystal belongs was reported by Kohn el al. to be $C_{2 h}{ }^{3}$, which is indeed the most probable one and with which all our data are compatible. There are $4 \mathrm{Ga}_{2} \mathrm{O}_{3}$ in the unit cell; ${ }^{4}$ the x-ray density is therefore $5.94 \mathrm{~g} / \mathrm{cm}$.
The crystal photographed with the Weissenberg camera and both $\mathrm{Cu} K \alpha$ and $\mathrm{Mo} K \alpha$ radiation had the dimensions: length 1.3 mm and cross section $0.09 \times 0.11$ mm . The diffraction symmetry was $C_{2 h}-2 / m$, with systematic absences, $h k l, h+k \neq 2 n$. Thus the possible space groups are $C_{2 h}{ }^{3}-\mathrm{C} 2 / \mathrm{m}, \mathrm{C}_{8}{ }^{3}-\mathrm{Cm}$ and $\mathrm{C}_{2}{ }^{3}-\mathrm{C} 2$. The space group Cm would require two equivalent atoms related by the symmetry plane to be at a maximum distance from each other of 1.52 A thus indicating that the crystal cannot possibly belong to this space group.
The corresponding intensities on the even-numbered (Weissenberg) layers about the $b$ axis were very similar as were those on the odd-numbered layers. Thus the structure appeared to be layered at one-half the $b$ axis. It was therefore unlikely that the heavy atoms would be at combinations of positions such that the $y$ coordinates differed by other than multiples of $\frac{1}{2} b$, (a)though, of course very small deviations might occur). In such a case the positions of $C 2$ with $y=0$ or $\frac{1}{2}$ become special positions of $C 2 / m$. Negative tests for piezoand pyroelectricity supported the conclusion that the most probable space group for the crystal is $C 2 / \mathrm{m}$.
In $C 2 / m$, the general positions and positions $e-h$ would require two equivalent atoms related by the symmetry plane to be at 1.52 A from each other. Thus atoms could not be in any of these positions.
The intensities were estimated visually by comparison with a calibrated intensity scale and by cross comparison of intensities on various photographs. Intensities of zero-level data taken with MoKa radiation were
averaged with those taken with $\mathrm{Cu} K \alpha$ radiation when observable with both radiations. (The small dispersion corrections for Ga would not have a significant effert.) Because no really new information was obtainable from other than the zero and first layers, intensities on these were the only ones quantitatively estimated.

Because absorption of both radiations by the crystal was high ( $\mu_{\mathrm{CuK} \mathrm{\alpha}}=299 \mathrm{~cm}^{-1}, \mu_{\mathrm{MoK} \mathrm{\alpha}}=255 \mathrm{~cm}^{-1}$ ), it was necessary to use a small crystal (see above). Thus maximum exposure times for the photographs taken with $\mathrm{Mo} \mathrm{K} \alpha$ radiations were $102,90,70$, and 90 hr for the zero through third layers, respectively. With $\mathrm{Cu} K \alpha$ radiation, the maximum exposure times were about 20 hr. Absorption corrections were applied assuming the crystal to be cylindrical with an average radius of 0.05 mm . The absorption correction tables used were those of Bond ${ }^{12}$ and applied to the first layer in accordance with the formula given by Bond. ${ }^{12}$ (see also footnote reference 13). Lorentz-polarization-Tunell rotationfactor corrections were also applied. ${ }^{14}$

A Patterson projection on (010) clearly showed the peaks resulting from $\mathrm{Ga}-\mathrm{Ga}$ interactions, and it appeared that all atoms must lie in the symmetry planes, and occupy five sets of positions $4 i$ : ( 000 , $\left.\frac{11}{2} 0\right) \pm(x 0 z)$. It was also clear that the contributions from the $\mathrm{Ga}^{3+}$ ions would determine most of the phases. It was necessary to consider the first-layer data for the purpose of distinguishing the most probable among homomorphic (010) projections. A (010) Fourier synthesis, a generalized Patterson projection using the $h 1 l$ data, a generalized Fourier projection with the $h 1 l$ data, and a pseudo-three-dimensional difference synthesis aided in determining the oxygen-ion positions. The pseudo-three-dimensional synthesis was done as follows: The contributions of the $\mathrm{Ga}^{3+}$ ions were subtracted from the observed structure factors. These had been scaled on the basis that, as indicated, the contributions of oxygens were generally not large. Thus some intuition entered into this scaling and as will be seen appeared to be quite sound. The remainders assumed to be the oxygen contributions, but of course containing numerous error contributions, were used in the synthesis. Each such amplitude, $h 0 l$, and $h 1 l$ was counted once in the synthesis. As expected, some spurious peaks occurred. Nevertheless, having obtained fairly accurate positions of the $\mathrm{Ga}^{3+}$ ions, the results of this synthesis together with structural considerations led to a determination of the oxygen trial parameters.

The trial parameters (for refinement) are given in Table I. It should be mentioned that the oxygen-ion positions were expected to be quite inaccurate, the idea being to save the author time by allowing the IBM 704 computer to do all of the refining.

[^2][^3]Table I. Trial parameters.

|  |  |  |  |
| :---: | :---: | :---: | :---: |
| Atom | Coordinates |  | $B, \mathrm{~A}^{2}$ |
|  | $x$ | $z$ |  |
|  |  |  |  |
| $\mathrm{Ga}_{\text {I }}$ | 0.087 | -0.204 | 0.2 |
| $\mathrm{Ga}_{\text {II }}$ | 0.341 | -0.312 | 0.2 |
| $\mathrm{O}_{\text {I }}$ | 0.153 | 0.092 | 0.8 |
| $\mathrm{O}_{\text {II }}$ | 0.492 | 0.248 | 0.8 |
| $\mathrm{O}_{\text {III }}$ | 0.829 | 0.443 | 0.8 |

## REFINEMENT OF THE STRUCTURE

Refinement of the structure was carried out with the Busing-Levy ${ }^{15}$ least-squares program for the IBM 704 computer. Only $h 0 l$ and $h 1 l$ amplitudes were used. In the first refinement cycles 824 data were included. All observed amplitudes were weighted unity; all unobserved amplitudes included at one-half threshold value were weighted 0.01 . Two scale factors corresponding to the two $k$ values (zero and first layers about $b$ ) of the data were refined. Both of these were started at 1.000 and are the scale factors mentioned earlier. Atomic scatcering factors used for the structure amplitude calculations were those of Thomas and Umeda ${ }^{16}$ for $\mathrm{Ga}^{3+}$ and for $\mathrm{O}^{2-}$ those given by Berghuis et al. ${ }^{17}$ for O arbitrarily modified such that $f_{0^{2}-}=10$ at $\sin \theta / \lambda=0$ and $f_{\mathrm{O}^{2-}}=f_{\mathrm{O}}$ at $\sin \theta / \lambda \geq 0.20$.

Convergence was attained within three.cycles: in the final cycle, the largest change in any coordinate was 0.003 A by $\mathrm{O}_{\text {II }}$ in the $a$ direction. The weighted $R$ factor was 0.171 for the data calculated on the basis of the parameters of the second cycle. The final scale factors were 0.988 and 0.997 with $\sigma$ 's of 0.010 , meaning that these are not significantly different from 1.000 .

Examination of the calculated and observed data indicated some large discrepancies in the high-angle data taken with the $\mathrm{Mo} K \alpha$ radiation. These data had been taken with Kodak type $K K$ film, which unfortunately has rather large grain size. Small weak spots are difficult to see in this case; the difficulty is more pronounced when the $\alpha_{1}, \alpha_{2}$ doublet is resolved. Because the data were plentiful, it was decided to omit a large portion of the high-angle reflections. The total data were therefore reduced from 824 to 522 . Two least-squares cycles were run. The largest difference in coordinates between the results so obtained and those obtained previously was 0.005 A for atom $\mathrm{O}_{\text {III }}$ in the $a$ direction. The final parameters with their standard deviations are given in Table II.

The weighted $R$ factor calculated by the BusingLevy program was 0.166 . However, if unobserved reflections are excluded; the $R$ factor

$$
\sum\left|F_{\text {obs }}-F_{\text {cale }}\right| /\left|\sum\right| F_{\text {obs }} \mid
$$

[^4]for the $h \mathrm{Ol}$ reflections is 0.143 and for the $h 1 l, 0.130$. The comparison of calculated and observed structure amplitudes is given in Table III.

## INTERATOMIC DISTANCES AND ANGLES

The estimated standard deviations of the positions of the atoms are $0.004,0.004,0.033,0.028$, and 0.028 A for $\mathrm{Ga}(\mathrm{I}), \mathrm{Ga}(\mathrm{II}), \mathrm{O}(\mathrm{I}), \mathrm{O}(\mathrm{II})$, and $\mathrm{O}(\mathrm{III})$, respectively. These standard deviations of position should be considered as radii of circles in the mirror planes. Standard deviations in bond lengths were computed taking into consideration the angle made between the bond direction and the symmetry planes. The interatomic distances and their standard deviations are given in Table IV.

The important angles are given in Table V. Individual standard deviations were not calculated for these, but an estimate of the standard deviation of an $\mathrm{O}-\mathrm{Ga}-0$ angle is $1.3^{\circ}$ and of a $\mathrm{Ga}-\mathrm{O}-\mathrm{Ga}$ angle, $0.9^{\circ}$.

## DESCRIPTION OF THE STRUCTURE

The arrangement of the ions in the $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ structure is shown in Figs. 1 and 2. The oxygen ions are arranged in a "distorted cubic" close-packed array. There are two crystallographically nonequivalent gallium and three nonequivalent oxygen ions in the unit cell. Each $\mathrm{Gar}^{3+}$ ion is surrounded by a distorted tetrahedron of oxygen ions: one $\mathrm{O}_{\mathrm{I}}{ }^{2-}$ at 1.80 A , two $\mathrm{O}_{\mathrm{HI}}{ }^{2-}$ ions at 1.83 A , and one $\mathrm{O}_{\mathrm{II}}{ }^{2-}$ ion at 1.85 A (average 1.83 A ). Each $\mathrm{Ga}_{\text {II }}{ }^{3+}$ ion is surrounded by a highly distorted octahedron of oxygen ions: two $\mathrm{O}_{\mathrm{I}}{ }^{2-}$ at 1.95 A , one $\mathrm{O}_{\mathrm{II}}{ }^{2-}$ ion at 1.95 A , one $\mathrm{O}_{\mathrm{III}}{ }^{2-}$ ion at 2.02 A , and two $\mathrm{O}_{\mathrm{III}}{ }^{2-}$ ions at 2.08 A (average 2.00 A ). A tetrahedron shares only corners with other tetrahedra in the $b$-axis direction and with octahedra in other directions. An octahedron shares edges with adjacent octahedra in the $b$ axis direction and in roughly the [102] direction: the shared edges are, respectively, $\mathrm{O}_{\mathrm{I}^{-}} \mathrm{O}_{\mathrm{III}}$ and $\mathrm{O}_{\mathrm{III}^{-}}$ $\mathrm{O}_{\text {III }}$, each of length 2.67 (Table IV). The distance 2.67 A is the shortest $\mathrm{O}-\mathrm{O}$ distance in the structure. This is

Table II. Final parameters.

|  | Scale factors |  | Standard deviations |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 10.984 |  | 0.013 |  |  |
|  |  | 20.988 |  | 0.014 |  |  |
| Atom | Coordinates |  | $B, \mathrm{~A}^{2}$ | $\sigma(x)$ | $\sigma(z)$ | $\sigma(B)$ |
|  | $x$ | $z$ |  |  |  |  |
| $\mathrm{Ga}_{1}$ | 0.0904 | -0.2052 | 0.33 | 0.0002 | 0.0005 | 0.05 |
| GaII | 0.3414 | -0.3143 | 0.28 | 0.0002 | 0.0005 | 0.05 |
| $\mathrm{O}_{\mathrm{I}}$ | 0.1674 | 0.1011 | 0.76 | 0.0019 | 0.0041 | 0.32 |
| $\mathrm{O}_{\text {II }}$ | 0.4957 | 0.2553 | 0.43 | 0.0016 | 0.0034 | 0.25 |
| $\mathrm{O}_{\text {III }}$ | 0.8279 | 0.4365 | 0.46 | 0.0015 | 0.0034 | 0.25 |

Table III. Calculated and observed amplitudes.


Table IV. Interatomic distances and standard deviations.

|  | Atom pair (No.) | Distance, A | $\sigma, \mathrm{A}$ |  | Atom pair (No.) | Distance, A | $\sigma, A$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Ga}_{1} \mathrm{O}_{4}$ tetrahedron | $\mathrm{Ga}-\mathrm{O}_{\mathrm{I}}$ | 1.80 | 0.03 | $\mathrm{Ga}_{11} \mathrm{O}_{6}$ octahedron | $\mathrm{Ga}-\mathrm{O}_{\mathrm{I}}(2)$ | 1.95 | 0.03 |
|  | $\mathrm{Ga}-\mathrm{O}_{11}(2)$ | 1.83 | 0.01 |  | $\mathrm{Ga}-\mathrm{O}_{11}$ | 1.95 | 0.03 |
|  | $\mathrm{Ga}-\mathrm{O}_{\mathrm{III}}$ | 1.85 | 0.03 |  | $\mathrm{Ga}-\mathrm{O}_{\mathrm{III}}$ | 2.02 | 0.03 |
|  | $\mathrm{O}_{\mathrm{I}}-\mathrm{O}_{11}(2)$ | 2.93 | 0.04 |  | $\mathrm{Ga}-\mathrm{O}_{\mathrm{III}}(2)$ | 2.08 | 0.02 |
|  | $\mathrm{O}_{\mathrm{I}}-\mathrm{O}_{\text {III }}$ | 3.13 | 0.04 |  | $\mathrm{O}_{\mathrm{I}}-\mathrm{O}_{\mathrm{I}}$ | 3.04 | 0.01 |
|  | $\mathrm{O}_{\mathrm{II}}-\mathrm{O}_{\text {II }}$ | 3.04 | 0.01 |  | $\mathrm{O}_{\mathrm{I}}-\mathrm{O}_{\mathrm{II}}(2)$ | 2.90 | 0.04 |
|  | $\mathrm{O}_{\mathrm{II}}-\mathrm{O}_{\mathrm{III}}(2)$ | 3.02 | 0.03 |  | $\mathrm{O}_{\mathrm{I}}-\mathrm{O}_{\mathrm{III}}(2)$ | 2.85 | 0.04 |
|  |  |  |  |  | $\mathrm{O}_{\mathrm{I}}-\mathrm{O}_{\mathrm{III}}(2)$ | 2.67 | 0.04 |
|  |  |  |  |  | $\mathrm{O}_{\mathrm{II}}-\mathrm{O}_{\mathrm{III}}(2)$ | 2.89 | 0.03 |
|  |  |  |  |  | $\mathrm{O}_{\mathrm{III}}-\mathrm{O}_{\text {III }}(2)$ | 2.67 | 0.04 |
|  |  |  |  |  | $\mathrm{O}_{\mathrm{III}}-\mathrm{O}_{\mathrm{III}}$ | 3.04 | 0.01 |
| Shortest Ga-Ga distances | $\mathrm{Ga}_{1}-\mathrm{Ga}_{1}(2)$ | 3.04 | 0.01 | Averages | $\mathrm{Ga}_{1}-\mathrm{O}$ | 1.83 |  |
|  | $\therefore \mathrm{Ga}_{11}-\mathrm{Ga}_{11}(2)$ | 3.04 | 0.01 |  | $\mathrm{Ga}_{\text {II }}-\mathrm{O}$ | 2.00 |  |
|  | - $\mathrm{Ga}_{\text {II }}-\mathrm{Ga}_{\text {II }}(2)$ | 3.11 | 0.01 |  | $\mathrm{O}-\mathrm{O}$, octa- | 2.84 |  |
|  | $\mathrm{Ga}_{1}-\mathrm{Ga}_{11}$ | 3.28 | 0.01 |  | hedron |  |  |
|  | $\mathrm{Ga}_{1}-\mathrm{Ga}_{\text {II }}(2)$ | 3.30 | 0.01 |  | $0-0$, tetra- | 3.02 |  |
|  | $\mathrm{Ga}_{1}-\mathrm{Ga}_{\text {II }}(2)$ | 3.33 | 0.01 |  | hedron |  |  |
|  | $\mathrm{Ga}_{\mathrm{I}}-\mathrm{Ga}_{\text {II }}(2)$ | 3.45 | 0.01 |  |  |  |  |

Table V. Bond angles.

${ }^{\mathbf{a}}$ GaI and $\mathrm{GaII}^{\text {I }}$ in same plane.
in agreement with the observation ${ }^{18}$ that in ionic structures, the mutual repulsion of the positive ions tends to reduce the length of shared edges of anion polyhedra.

Because of the short $b$ axis, there are two $\mathrm{O}_{\mathrm{I}}{ }^{2-}$ and two $\mathrm{O}_{\mathrm{III}}{ }^{2-}$ ions (along the $b$ axis) at corners of an octahedron. The structure cannot possibly then have two $\mathrm{O}_{\mathrm{HI}}{ }^{2-}$ ions at the remaining corners of the octahedron, since these must lie in the mirror plane containing the $\mathrm{Ga}_{11}{ }^{3+}$ ion within the octahedron. Thus there is only one $\mathrm{O}_{\mathrm{II}}{ }^{2-}$ ion at a corner of the octahedron, the remaining corner being occupied by a third $\mathrm{O}_{\mathrm{II}}{ }^{2-}$ ion.

At the corners of the tetrahedron, there are two $\mathrm{O}_{\mathrm{II}}{ }^{2-}$ ions which are along the $b$ axis, the other corners

[^5]being occupied by an $\mathrm{O}_{\mathrm{I}}{ }^{2-}$ and an $\mathrm{O}_{\mathrm{HI}^{2-}}$ ion each lying in the mirror plane containing the $\mathrm{Ga}^{3+}$ ion within the tetrahedron.
Thus eách $\mathrm{O}_{\mathrm{I}}{ }^{2-}$ ion is at the corner of two octahedra and one tetrahedron; each $\mathrm{O}_{\mathrm{II}}{ }^{2-}$ ion is at the corner of one octahedron and two tetrahedra; and each $\mathrm{OHII}^{2-}$ ion is at the corner of three octahedra and one tetrahedron. If the octahedra and tetrahedra were regular, it would be doubtful that such a structure could exist, because the sums of the bond numbers of the bonds at all oxygen ions would not be 2 (see footnote reference 18). They would be: at $\mathrm{O}_{1}{ }^{2-}, 1 \frac{3}{4}$, at $\mathrm{O}_{\mathrm{Ir}^{2-}}{ }^{2}, 2$; and at $\mathrm{O}_{\mathrm{HI}}{ }^{2-}, 2 \frac{1}{4}$. However, the polyhedra are probably not regular. In fact, the four bonds to $\mathrm{O}_{\mathrm{II}}{ }^{2-}$ are the longest ones: $\mathrm{Ga}_{\mathrm{I}}-\mathrm{O}_{\mathrm{III}}=1.85 \mathrm{~A}, \mathrm{Ga}_{\mathrm{II}}-\mathrm{O}_{\mathrm{III}}=2.08$ (2) and 2.02

A, whereas those to $\mathrm{O}_{\mathrm{I}}{ }^{2-}$ are the shortest ones: $\mathrm{Ga}_{\mathrm{I}}-\mathrm{O}_{\mathrm{I}}=$ 1.80 A and $\mathrm{Ga}_{\text {II }}-\mathrm{O}_{\mathrm{I}}=1.95 \mathrm{~A}(2)$. The $\mathrm{Ga}_{\mathrm{I}}-\mathrm{O}_{\mathrm{II}}$ distance 1.83 A is equal to the average $\mathrm{Ga}_{1}-\mathrm{O}$ distance, but the $\mathrm{Ga}_{\text {II }}-\mathrm{O}_{\text {II }}$ distance, 1.95 A , is 0.05 A shorter than the average $\mathrm{Ga}_{\text {II }}-\mathrm{O}$ distance. Thus the sum of the bond numbers of the bonds to an $\mathrm{O}_{\mathrm{HI}}{ }^{2-}$ ion is somewhat greater than two, but not importantly so (see footnote reference 20 ).

Actually, the statistical calculation tells us that: there are no significant differences in $\mathrm{Ga}_{1}-\mathrm{O}$ distances, nor in the $\mathrm{Ga}_{\text {II }^{-}-\mathrm{O}_{\mathrm{I}} \text { and } \mathrm{Ga}_{\text {II }}-\mathrm{O}_{\text {II }} \text { distances, nor in the }}$ two $\mathrm{Ga}_{\text {II }}-\mathrm{O}_{\text {III }}$ distances. The shorter $\mathrm{Ga}_{\text {II }}-\mathrm{O}_{\text {III }}$ distance is only possibly significantly different from the $\mathrm{GaII}^{-}$ $\mathrm{O}_{\mathrm{I}}$ and $\mathrm{Ga}_{\text {II }}-\mathrm{O}_{\text {II }}$ distances. However the long $\mathrm{Ga}_{\text {II }}-\mathrm{O}_{\text {III }}$ distance is significantly larger than theshortest distances. Unfortunately, the oxygen ions contribute little to the intensities in comparison with gallium ions and it is unlikely that much greater reliability can be attained even from more accurate intensity data. Also it would appear to be quite difficult to obtain crystals of the aluminum isomorph designated as $\theta-\mathrm{Al}_{2} \mathrm{O}_{3}{ }^{19,20}$; none have as yet been reported.

Nevertheless, the aforementioned criterion defined by Pauling ${ }^{18}$ indicates that the oxygen polyhedra must be irregular and therefore that the calculated distances are perhaps better than indicated by the standard deviations.

## DISCUSSION

## 1. Comparison with $\alpha$-Corundum Structure

The $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ structure (Figs. 1 and 2) appears to be quite different from that of $\alpha-\mathrm{Ga}_{2} \mathrm{O}_{3}$ which has the $\alpha$ corundum structure. The latter has the oxygen ions in approximately hexagonal close-packed array with all the $\mathrm{Ga}^{3+}$ ions octahedrally coordinated to $\mathrm{O}^{2-}$ ions. Also in the $\alpha$ phase, octahedra share edges and faces


Fig. 1. Perspective view of the arrangement of the oxygen octahedra and tetrahedra in $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$.

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Fig. 2. Plan view of the $\beta$ - $\mathrm{Ga}_{2} \mathrm{O}_{3}$ structure. Lines connecting atoms indicate the manner in which the octahedra and tetrahedra are constructed from and joined by the oxygen atoms in the three consecutive symmetry planes. Only the upper faces of the polyhedra are depicted. For increased clarity, this figure should be studied simultaneously with Fig. 1.
which brings the metal ions very near each other. Accurate atomic positional parameters for $\alpha-\mathrm{Ga}_{2} \mathrm{O}_{3}$ have not been determined. However, in $\mathrm{Fe}_{2} \mathrm{O}_{3},{ }^{1}$ the closest approach of two $\mathrm{Fe}^{3+}$ ions through a shared octahedral face is 2.88 A and through a shared edge $2.96 \mathrm{~A} .{ }^{21}$ In $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ no faces are shared between polyhedra and the shortest $\mathrm{Ga}^{3+}-\mathrm{Ga}^{3+}$ distance is 3.04 A .

Now it is recognized that usually structures in which faces of polyhedra are shared are less stable than those in which edges are shared, which in turn are less stable than structures in which only corners are shared. ${ }^{18}$ Thus, one would expect the $\beta$ phase to be more stable than the $\alpha$ phase. Foster and Stumpf have shown ${ }^{10}$ that although the $\alpha-\mathrm{Ga}_{2} \mathrm{O}_{3}$ forms at lower temperatures than does $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$, the $\alpha$ phase is metastable. In the case of alumina, it appears that the $\alpha$ phase forms at higher temperatures than the $\theta$ (isostructural with $\beta$ $\mathrm{Ga}_{2} \mathrm{O}_{3}$ ) just the reverse of the gallia. Yet it appears that although seen rarely in comparison with the $\alpha$ phase, the $\theta-\mathrm{Al}_{2} \mathrm{O}_{3}$ is the thermodynamically stable phase at room temperature ${ }^{19}$ and that although $\alpha-\mathrm{Al}_{2} \mathrm{O}_{3}$ is thermodynamically metastable at room temperature the $\alpha \rightarrow \theta$ transition of $\mathrm{Al}_{2} \mathrm{O}_{3}$ is "infinitely" sluggish at such temperature.

As one might expect, the lower average coordination in the $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ is accompanied by a lower density; the volumes per $\mathrm{Ga}_{2} \mathrm{O}_{3}$ in the $\alpha$ and $\beta$ phases are 47.8 and $52.8 \mathrm{~A}^{3}$, respectively.

[^7]Table VI. Analogous average distances and ranges in $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ and in yttrium-iron garnet.

|  | $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ |  | YIG ${ }^{\text {a }}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| Metal-oxygen distances | Averages | Range | Averages | Range |
| Octahedral Tetrahedral | $\begin{aligned} & 2.00 \mathrm{~A} \\ & 1.83 \end{aligned}$ | $\begin{aligned} & 1.95-2.08 \mathrm{~A} \\ & 1.80-1.85 \end{aligned}$ | $\begin{aligned} & 2.00 \mathrm{~A} \\ & 1.88 \end{aligned}$ | All equal <br> All equal |
| O-O distances |  |  |  |  |
| In octahedron In tetrahedron | $\begin{aligned} & 2.84 \\ & 3.02 \end{aligned}$ | $\begin{aligned} & 2.67-2.90 \\ & 2.93-3.13 \end{aligned}$ | $\begin{aligned} & 2.84 \\ & 3.06 \end{aligned}$ | $\underset{2.87-3.16}{2.68-2.99 \mathrm{~A}}$ |

${ }^{\text {a }}$ S. Geller and M. A. Gillco, J. Phys. Chem. Solids 3, 30 (1957); 9, 235 (1959).

## 2. Possibility of Disorder

That the structure is ordered appears to be established by the paramagnetic-resonance work by Peter and Schawlow ${ }^{2}$ on $\mathrm{Cr}^{3+}$-ion-doped $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$, in which it is found that the $\mathrm{Cr}^{3+}$ ions prefer only one set of octahedral sites. It is likely that these ions replace the $\mathrm{Ga}_{11}{ }^{3+}$ ions in octahedrally coordinated sites: There is not enough space for foreign ions in the remaining octahedrally coordinated holes. Thus it is improbable that there is the type of disorder which would allow drastic change in the environment of any of the $\mathrm{Ga}_{\mathrm{I}}{ }^{3+}$ and $\mathrm{Ga}_{\mathrm{II}^{3+}}{ }^{3+}$ ions.

## 3. Importance of $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ Structure Relative to Substitution of $\mathrm{Ga}^{3+}$ and $\mathrm{Al}^{3+}$ for $\mathrm{Fe}^{3+}$ Ions in Yttrium-Iron Garnet

The $\mathrm{Ga}^{3+}$ and $\mathrm{Fe}^{3+}$ ions have very similar crystal chemistry. Both ions have spherical electronic configuration and are of very nearly the same size, the $\mathrm{Ga}^{3+}$ ion being somewhat smaller than $\mathrm{Fe}^{3+}$ ion in most structures. The relative radii derived from the perovskitelike compounds ${ }^{8}$ put the $\mathrm{Ga}^{3+}$ ion $C N(6)$ radius at 0.015 A less than that of the $\mathrm{Fe}^{3+}$ ion. This does not mean that average metal-oxygen distances will be exactly the same in different structures. For example, the relative ionic radii derived from the perovskitelike compounds are applicable to the garnets, ${ }^{5,23,24}$ but the $C N(6)$ metal-oxygen distances in the garnets are uniformly somewhat larger than the sums of these radii.
Although the structure of yttrium-gallium garnet, $\left\{\mathrm{Y}_{3}\right\}\left[\mathrm{Ga}_{2}\right]\left(\mathrm{Ga}_{3}\right) \mathrm{O}_{12}$ has not yet been refined, that of yttrium-iron garnet has ${ }^{25}$; it is of interest to compare some analogous distances in this garnet structure with those of $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$. The averages and ranges of these analogous distances are compared in Table VI. It is seen that the average $\mathrm{Ga}_{11}-\mathrm{O}$ distance in $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ is the same as the octahedral $\mathrm{Fe}-\mathrm{O}$ distance in yttriumiron garnet. On the other hand, the average $\mathrm{Ga}_{\mathrm{I}}-\mathrm{O}$

[^8]distance in $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ is substantially smaller than the tetrahedral $\mathrm{Fe}-\mathrm{O}$ distance in the garnet. The average $\mathrm{O}-\mathrm{O}$ distances of the octahedra in the two structures are equal but the average $\mathrm{O}-\mathrm{O}$ distance in the $\mathrm{GaO}_{4}$ tetrahedron in $\beta$ - $\mathrm{Ga}_{2} \mathrm{O}_{3}$ is shorter than the average $\mathrm{O}-\mathrm{O}$ distance in the $\mathrm{FeO}_{4}$ tetrahedron of the garnet. Now from the least-squares calculations, the aforementioned differences are not significant. However, on a crystal chemical basis, there is reason to believe that the differences are meaningful.

We have shown previously ${ }^{5}$ that when the $\mathrm{Ga}^{3+}$ ion is substituted for the $\mathrm{Fe}^{3+}$ ion in the garnets, it shows a great preference for the tetrahedral $[C N(4)]$ site. Also there is indication that when $\mathrm{Fe}^{3+}$ is substituted for the $\mathrm{Ga}^{3+}$ ion in yttrium-gallium garnet, it greatly prefers the octahedral $[C N(6)]$ site. ${ }^{26}$ We have also mentioned elsewhere that the ratios of effective size of ions in different coordinations may differ and probably depend largely on the nuclear charge and external electronic configuration of the atom. ${ }^{23,24}$ The ratios of tetrahedral to octahedral $\mathrm{Fe}-\mathrm{O}$ distances in yttrium-iron garnet is 0.94 ; in $\mathrm{Y}_{3} \mathrm{Al}_{2} \mathrm{Al}_{3} \mathrm{O}_{12}{ }^{27}$ the analogous Al-O ratio is 0.91 , in $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ the ratio of tetrahedral to octahedral average $\mathrm{Ga}-\mathrm{O}$ distances is 0.91 , in fact very similar to that of the $\mathrm{Al}-\mathrm{O}$ distances in the aluminum garnet.

Now despite the fact that the $\mathrm{Al}^{3+}$ ion is much smaller than the $\mathrm{Ga}^{3+}$ ion, the quantitative site preference in the substituted iron garnets of the latter is very close to that of the former ${ }^{5,9}$ : this is in agreement with the above considerations.

## 4. Magnetic Aspects

If an $\mathrm{Fe}_{2} \mathrm{O}_{3}$ phase isostructural with $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ were found, it should be antiferromagnetic, because the $\mathrm{Ga}-\mathrm{O}-\mathrm{Ga}_{\text {II }}$ angles of about $123^{\circ}$ (Table V) are (and presumably the $\mathrm{Fe}_{\mathrm{I}}-\mathrm{O}-\mathrm{Fe}_{\text {II }}$ angles would be) favorable to superexchange interaction ${ }^{9,25,28,29}$ and the network of octahedra and tetrahedra involving these favorable angles continues throughout the structure. ${ }^{29}$ Counting the possible significant magnetic interactions we find

[^9]Table VII. $\beta$ - $\mathrm{Ga}_{2} \mathrm{O}_{3}$ powder data ( $\mathrm{Cu} K \alpha$ radiation).


* Beginning with this line the $\alpha_{1}, \alpha_{2}$ doublet on the powder photograph was resolved.
$\dagger \alpha_{2}$ Line from previous reflection overlaps $\alpha_{1}$ of this reflection.
that there are six interactions of a tetrahedrally coordinated ion with octahedrally coordinated ions and six interactions of an octahedrally coordinated ion with tetrahedrally coordinated ions. Thus there would be six important magnetic interactions per magnetic ion. From the recent paper by Gilleo, ${ }^{28}$ one would estimate a Néel temperature of about $700^{\circ} \mathrm{K}$ for an $\mathrm{Fe}_{2} \mathrm{O}_{3}$ phase isostructural with $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$.

Now, the facts that: (1) the $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ structure appears to be favorable to antiferromagnetic interaction, (2) that $\mathrm{Fe}^{3+}$ ion has a crystal chemistry similar
to that of $\mathrm{Ga}^{3+}$ ion, and (3) that the $\mathrm{Fe}^{3+}$ ion would prefer octahedral sites if substituted for $\mathrm{Ga}^{3+}$ in yttriumgallium garnet, all would immediately indicate the possibility of producing a ferrimagnetic material from solid solutions of $\mathrm{Fe}_{2} \mathrm{O}_{3}$ in $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$. However, experiments made by Remeika prior to the determination of this structure, indicated that not much $\mathrm{Fe}^{3+}$ ion could be made to dissolve in $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ by solid-solid reaction, ${ }^{30}$ not enough, that is, to produce strong enough interac-

[^10]Table VIII. Comparison of some relative diffractometer and calculated intensities.
$\left.\begin{array}{ccc}\hline \hline h k l & I_{0} & I_{0} \\ \hline 400 \\ 110\end{array}\right)$
tion. On the other hand, Remeika has discovered a new structure of formula $\mathrm{Ga}_{2}{ }_{x} \mathrm{Fe}_{x} \mathrm{O}_{3}$ which is ferrimagnetic (and also piezoelectric). ${ }^{3}$ Crystals have been made with composition $x=0.7$ to 1.4 , and these have been reported by Wood ${ }^{31}$ to belong to space group $C_{2 v}{ }^{9}-P c 2 n$ with eight formula units per cell. Thus there must be in this crystal four sets of metal ions in the general positions, the only positions, in the space group. Because the crystal with $x=1.0$ is still quite ferrimagnetic ( $n_{B}$ at $4.2^{\circ} \mathrm{K}$ and $H=\infty$ is 0.95$)^{3}$ the indication is that the metal ions in the structure show a site preference which gives the net moment. Because both $\mathrm{Fe}^{3+}$ and $\mathrm{Ga}^{3+}$ ions have spherical electronic configurations, and the difference in their $C N(6)$ radii is presumably much smaller than the difference in their tetrahedral radii, there is the strong implication that there are in the $\mathrm{Ga}_{2-x} \mathrm{Fe}_{x} \mathrm{O}_{3}$ two kinds of coordinations for the metal ions.

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[^11]
## APPENDIX

## $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ Powder Data

The data obtained from the x-ray diffraction powder pattern of pulverized $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ crystals taken with $\mathrm{Cu} \mathrm{K}_{\alpha}$ radiation are in good agreement with those of Kohn et al. ${ }^{4}$ However, there are some minor discrepancies. Furthermore, a knowledge of the structure allows us to index the pattern more adequately, although the indexing of the highest-angle lines may still be in some doubt.

In Table VII, the calculated spacings and intensities ${ }^{3 ?}$ are compared with those observed. The calculations were carried out with an IBM 704 program devised by Dr. R. G. Treuting. Several calculations of spacings with slightly different values of lattice constants indicated that the values given by Kohn et al. ${ }^{4}$ are good to within the limits of error specified. Calculations of spacings only were carried out with a separate IBM 704 program also devised by Dr. Treuting.

It will be noticed immediately that the observed intensities given are only qualitative. It would appear that quantitative intensity measurements taken with a diffractometer would be called for. Attempts were made to do this, but preferred orientation difficulties indicated that obtaining a proper pattern would indeed be a time-consuming project.

A finely divided powder all of which passed through a 400 -mesh sieve, was used as the specimen for the revolving specimen holder of the Norelco diffractometer. Examination of Table VIII, in which the intensity of the ( $\{400\},\{110\}$ ) reflection is taken as unity and several others are compared with it, is indicative of the extreme effect.

Examination of the powder specimen with a microscope ( $144 \times$ ) indicated that the crystallites were needle- or platelike with the needle axis or plate tending to lie flat. The rotating specimen photographed with the Norelco 114.6 cm camera indicated a much more random distribution of crystallites and gave qualitative intensities in much better agreement with the calculated ones.

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    ${ }^{14} \mathrm{I}$ am indebted to Dr. R. G. Treuting for programing the calculation of the corrected relative squared structure amplitudes and relative structure amplitudes on the IBM 704 computer.

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[^5]:    ${ }^{\text {is }}$ L. Pauling, Nature of the Chemical Bond (Cornell University Press, Ithaca, New York, 1960), 3rd ed., Chap. 13, Sec. 6.

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    ${ }^{20}$ The lattice constants of this phase derived by the present author from the powder data given by Stumpf et al. ${ }^{19}$ are $\iota=$ $11.83 \pm 0.02, b=2.92 \pm 0.01, c=5.64 \pm 0.01 \mathrm{~A}$, and $\beta=104.0 \pm$. $5^{\circ}$. The powder data, however, are not completely correct, the most outstanding discrepancy being that of $d_{001}$ which should be 5.47 A as against the reported 5.25 A . Also it is possible that some observable lines have been omitted; one of these is the [401] reflection.

[^7]:    ${ }^{21}$ Recent accurate work on $\mathrm{Ti}_{2} \mathrm{O}_{3}$ and $\mathrm{V}_{2} \mathrm{O}_{3}$ by Nordmark ${ }^{22}$ has led to the $\mathrm{Me}-\mathrm{Me}$ distances: 2.55 and 2.64 A , respectively, across the shared face and 2.99 and 2.88 A , respectively, across the shared edge. In any case, the closest approach of $\mathrm{Ga}^{3+}$ ions in $\beta-\mathrm{Ga}_{2} \mathrm{O}_{3}$ is substantially greater than that of the metal ions in any of the $\alpha$ phases.
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[^10]:    ${ }^{20}$ J. P. Remeika (private communication).

[^11]:    ${ }^{31}$ E. A. Wood (to be published).

[^12]:    ${ }^{32}$ The expression for the calculated relative intensities is $I_{c}=$ ( $p / 4$ ) $L P F^{2} \times 10^{-4}$ where $p$ is the multiplicity, $L$ the Lorentz factor, $P$ the polarization factor, and $F$ the structure amplitude.

